Hong Kong Mathematics Olympiad (2007 – 2008)

Final Event 1 (Individual)

香港数学竞赛 (2007 – 2008)

决赛项目 1 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 设直线 $A = 15 \times \tan 44° \times \tan 45° \times \tan 46°$，求 A 的值。

Let $A = 15 \times \tan 44° \times \tan 45° \times \tan 46°$, find the value of A.

2. 设 n 为正整数及 $\overbrace{20082008\cdots2008}^{n\text{个 }2008}15$ 能被 A 整除。若 n 的最小可能值是 B，求 B 的值。

Let n be positive integer and $\overbrace{20082008\cdots2008}^{n\ 2008\text{'s}}15$ is divisible by A. If the least possible value of n is B, find the value of B.

3. 已知有 C 个整数满足方程 $|x-2|+|x+1| = B$，求 C 的值。

Given that there are C integers that satisfy the equation $|x-2|+|x+1| = B$, find the value of C.

4. 在坐标平面上，点 $(-C, 0)$ 与直线 $y = x$ 的距离是 $\sqrt{D}$，求 D 的值。

In the coordinate plane, the distance from the point $(-C, 0)$ to the straight line $y = x$ is $\sqrt{D}$, find the value of D.

Final Event 2 (Individual)

香港数学竞赛 (2007 – 2008)

决赛项目 2 (个人)

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1. 设 $\sqrt{a}=\sqrt{7+\sqrt{13}}-\sqrt{7-\sqrt{13}}$，求 P 的值。

Given that $P=\left[\sqrt[3]{6}\times\left(\sqrt[3]{\frac{1}{162}}\right)\right]^{-1}$, find the value of P.

2. 设 a、b 和 c 是实数且 $b:(a+c)=1:2$ 及 $a:(b+c)=1:P$。若 $Q=\frac{a+b+c}{a}$，求 Q 的值。

Let a, b and c be real numbers with ratios $b:(a+c)=1:2$ and $a:(b+c)=1:P$. If $Q=\frac{a+b+c}{a}$, find the value of Q.

3. 设 $R=\left(\sqrt{\sqrt{3}+\sqrt{2}}\right)^{Q}+\left(\sqrt{\sqrt{3}-\sqrt{2}}\right)^{Q}$，求 R 的值。

Let $R=\left(\sqrt{\sqrt{3}+\sqrt{2}}\right)^{Q}+\left(\sqrt{\sqrt{3}-\sqrt{2}}\right)^{Q}$, find the value of R.

4. 设 $S=(x-R)^2+(x+5)^2$，其中 x 为实数，求 s 的最小值。

Let $S=(x-R)^2+(x+5)^2$, where x is a real number. Find the minimum value of S.

Hong Kong Mathematics Olympiad (2007 – 2008)

Final Event 3 (Individual)

香港数学竞赛 (2007 – 2008)

决赛项目 3 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 $\frac{1-\sqrt{3}}{2}$ 满足方程 $x^2+px+q=0$，其中 p 和 q 是有理数。若 $A=|p|+2|q|$，求 A 的值。

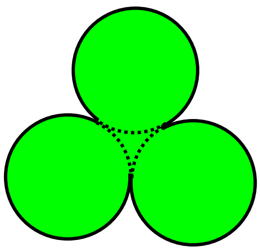
Given that $\frac{1-\sqrt{3}}{2}$ satisfies the equation $x^2+px+q=0$, where p and q are rational numbers. If $A=|p|+2|q|$, find the value of A.

2. U_1 及 U_2 两代有大小相同的红球和白球。U_1 装有 A 个红球，2 个白球。U_2 装有 2 个红球，B 个白球。从每袋中各取出两个球。若取到四个红球的概率是 $\frac{1}{60}$，求 B 的值。

Two bags U_1 and U_2 contain identical red and white balls. U_1 contains A red balls and 2 white balls. U_2 contains 2 red balls and B white balls. Take two balls out of each bag. If the probability of all four balls are red is $\frac{1}{60}$, find the value of B.

3. 图一由三个大小相同且互切的圆所组成，三个圆的半径均是 B cm。若阴影部分的周界是 C cm，求 C 的值。(取 $\pi=3$)

Figure 1 is formed by three identical circles touching one another, the radius of each circle is B cm. If the perimeter of the shaded region is C cm, find the value of C. (Take $\pi=3$)



图一

Figure 1

4. 设与 $\sqrt{C}$ 最接近的整数是 D， 求 D 的值。

Let D be the integer closet to $\sqrt{C}$, find the value of D .

Hong Kong Mathematics Olympiad (2007 – 2008)

Final Event 4 (Individual)

香港数学竞赛 (2007 – 2008)

决赛项目 4 (个人)

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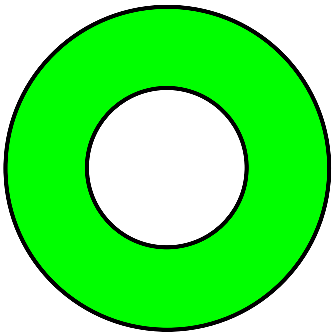
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 x 及 y 为实数，且满足 $|x|+x+y=10$ 及 $|y|+x-y=10$。若 $P=x+y$，求 P 的值。

Given that x and y are real numbers such that $|x|+x+y=10$ and $|y|+x-y=10$. If $P=x+y$, find the value of P.

2. 如图一，阴影部分由两同心圆所组成，其面积为 96π cm^2。若该两圆的半径相差 $2P$ cm 及大圆的面积为 Q cm^2，求 Q 的值。

In Figure 1, the shaded area is formed by two concentric circles and has area 96π cm^2. If the two radii differ by $2P$ cm and the large circle has area Q cm^2, find the value of Q. (Take $\pi = 3$)



图一

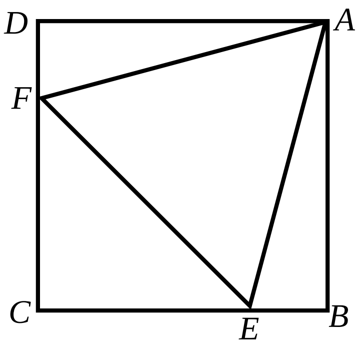
Figure 1

3. 设 R 为最大的整数使得 $R^Q < 5^{200}$，求 R 的值。

Let R be the largest integer such that $R^Q < 5^{200}$, find the value of R.

4. 图二显示一边长为 $(R-1)$ cm 的正方形 $ABCD$ 及一等边三角形 AEF (E 及 F 分别是直线 BC 及 CD 上的点)。 若 ΔAEF 的面积是 $(S-3)$ cm^2，求 S 的值。

In Figure 2 , there are a square $ABCD$ with side length $(R-1)$ cm and an equilateral triangle AEF (E and F are points on BC and CD respectively). If the area of ΔAEF is $(S-3)$ cm^2 , find the value of S .



图二

Figure 2